Physics I
Mr. Chou

Momentum

I. Definition of Momentum
Momentum is the quantity of motion an object has. The momentum of an object is defined by the equation of momentum = mass times velocity or \( mv \). Momentum is a vector (vector/scalar) quantity. Since the mass of an object is always positive, the direction of the momentum vector is always the same as the direction of the velocity vector. The direction of the velocity vector is always the same as the direction of motion (which way the object is currently heading).

II. Comparison with Kinetic Energy

<table>
<thead>
<tr>
<th></th>
<th>Momentum</th>
<th>Kinetic Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depends on</td>
<td>Mass</td>
<td>Mass</td>
</tr>
<tr>
<td></td>
<td>velocity</td>
<td>speed</td>
</tr>
<tr>
<td>Vector or Scalar</td>
<td>vector</td>
<td>scalar</td>
</tr>
<tr>
<td>Used in analyzing</td>
<td>collisions</td>
<td>Single or multiple objects</td>
</tr>
</tbody>
</table>

Given that \( p = mv \), momentum would have units of \( \text{kg m/s} \).

Law of Conservation of Momentum

Assume no external forces (friction, applied force, etc) act on a system of two or more objects. This situation is also known as an isolated system. Then the total momentum in a collision is conserved. That is, the total momentum stays constant. The momentum one object loses must be gained by some other object in the collision.

Problem solving for collisions:
1) A 15-kg medicine ball is thrown at a velocity of 20 km/hr to a 60-kg person who is at rest on ice. The person catches the ball and subsequently slides with the ball across the ice. Determine the velocity of the person and the ball after the collision.

\[
\text{Momentum} = \text{mass} \times \text{velocity} \quad \text{(kg m/s)}
\]

<table>
<thead>
<tr>
<th></th>
<th>Before collision</th>
<th>After collision</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 15 \times 20 = 300 )</td>
<td>( 15 , v_f )</td>
</tr>
<tr>
<td>ball</td>
<td>0</td>
<td>( 60 , v_f )</td>
</tr>
<tr>
<td>person</td>
<td>0</td>
<td>( 60 , v_f )</td>
</tr>
<tr>
<td>Total momentum</td>
<td>300 kg km/hr</td>
<td>300 kg km/hr</td>
</tr>
</tbody>
</table>

\[
75 \, v_f = 300 \\
v_f = 300/75 = 4 \, \text{km/hr}
\]
2) A 3000-kg truck moving with a velocity of 10 m/s hits a 1000-kg parked car. The impact causes the 1000-kg car to be set in motion at 15 m/s. Assuming that friction effects from the road are negligible, determine the velocity of the truck immediately after the collision.

\[
\text{Momentum} = \text{mass} \times \text{velocity} \quad (\text{kg} \, \text{m/s})
\]

<table>
<thead>
<tr>
<th></th>
<th>Before collision</th>
<th>After collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>truck</td>
<td>3000*10 = 30000</td>
<td>3000 ( v_f )</td>
</tr>
<tr>
<td>car</td>
<td>0</td>
<td>1000 * 15 = 15000</td>
</tr>
<tr>
<td><strong>Total momentum</strong></td>
<td><strong>30000 kg m/s</strong></td>
<td></td>
</tr>
</tbody>
</table>

\[
30000 = 3000 \, v_f + 15000
\]

\[
3000 \, v_f = 15000
\]

\[
v_f = 5 \, \text{m/s}
\]

3) A 3000-kg truck moving rightward with a speed of 5 km/hr collides head-on with a 1000-kg car moving leftward with a speed of 10 km/hr. The two vehicles stick together and move with the same velocity after the collision. Determine the post-collision velocity of the car and truck. (CAREFUL: Be cautious of the +/- sign on the velocity of the two vehicles.)

\[
\text{Momentum} = \text{mass} \times \text{velocity} \quad (\text{kg} \, \text{m/s})
\]

<table>
<thead>
<tr>
<th></th>
<th>Before collision</th>
<th>After collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>truck</td>
<td>3000*5 = 15000</td>
<td>3000 * ( v_f )</td>
</tr>
<tr>
<td>car</td>
<td>1000* (-10) = -10000</td>
<td>1000 * ( v_f )</td>
</tr>
<tr>
<td><strong>Total momentum</strong></td>
<td><strong>5000 kg km/hr</strong></td>
<td><strong>4000 ( v_f )</strong></td>
</tr>
</tbody>
</table>

\[
5000 = 4000 \, v_f
\]

\[
v_f = 1.25 \, \text{km/hr}
\]
Collision Analysis

Read from Lesson 2 of the Momentum and Collisions chapter at The Physics Classroom:
http://www.physicsclassroom.com/Class/momentum/u4L2d.html
http://www.physicsclassroom.com/Class/momentum/u4L2e.html

MOP Connection: Momentum and Collisions: sublevels 7, 8 and 9

1. A 10-kg medicine ball is thrown at a velocity of 15 km/hr to a 50-kg skater who is at rest on ice. The skater catches the ball and subsequently slides with the ball across the ice. Consider the skater and the ball as two separate parts of an isolated system. Fill in the before- and after-collision table below.

<table>
<thead>
<tr>
<th>Before Collision</th>
<th>Momentum After Collision</th>
<th>Momentum Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball</td>
<td>10 x 15 = 150</td>
<td>10 x V = 10V</td>
</tr>
<tr>
<td>Skater</td>
<td>50 x 0 = 0</td>
<td>50V</td>
</tr>
<tr>
<td>Total</td>
<td>150 + 0 = 150</td>
<td>10V + 50V = 60V</td>
</tr>
</tbody>
</table>

Determine the velocity of medicine ball and the skater after the collision. PSYW

\[ V = \frac{150}{60} = 2.5 \, \text{m/s} \]

2. A large fish with a mass of 1-kg is in motion at 45 cm/s when it encounters a smaller fish (m=0.25 kg) which is at rest. The large fish swallows the smaller fish and continues in motion at a reduced speed. Fill in the before- and after-collision table below.

<table>
<thead>
<tr>
<th>Before Collision</th>
<th>Momentum After Collision</th>
<th>Momentum Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Fish</td>
<td>1 x 45 = 45</td>
<td>1 x V = 1V</td>
</tr>
<tr>
<td>Small Fish</td>
<td>0.25 x 0 = 0</td>
<td>0.25 x V = 0.25V</td>
</tr>
<tr>
<td>Total</td>
<td>45</td>
<td>1V + 0.25V = 1.25V</td>
</tr>
</tbody>
</table>

Determine the velocity of the large and the small fish after the collision. PSYW

\[ V = \frac{45}{1.25} = 36 \, \text{cm/s} \]
3. A 0.150-kg baseball moving at a speed of 45.0 m/s crosses the plate and strikes the 0.250-kg catcher's mitt (originally at rest). The catcher's mitt immediately recoils backwards (at the same speed as the ball) before the catcher applies an external force to stop its momentum. If the catcher's hand is in a relaxed state at the time of the collision, it can be assumed that no net external force exists and the law of momentum conservation applies to the baseball-catcher's mitt collision. Fill in the before- and after-collision table below.

<table>
<thead>
<tr>
<th></th>
<th>Momentum Before Collision</th>
<th>Momentum After Collision</th>
<th>Momentum Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball</td>
<td>0.15 \times 45 = 6.75</td>
<td>0.15 \ V</td>
<td></td>
</tr>
<tr>
<td>Catcher's Mitt</td>
<td>0.25 \times 0 = 0</td>
<td>0.25 \ V</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>6.75</td>
<td>0.40 \ V</td>
<td></td>
</tr>
</tbody>
</table>

Determine the velocity of the baseball/catcher's mitt immediately after the collision. PSYW

\[
\frac{6.75}{0.40} = \frac{v}{v} \Rightarrow v = 16.9 \text{ m/s}
\]

4. A 4800-kg truck traveling with a velocity of +4.0 m/s collides head-on with a 1200-kg car have traveling with a velocity of -12 m/s. The truck and car entangle and move together after the collision. Fill in the before- and after-collision table below.

<table>
<thead>
<tr>
<th></th>
<th>Momentum Before Collision</th>
<th>Momentum After Collision</th>
<th>Momentum Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>4800 \times 4 = 19,200</td>
<td>4800 \ V</td>
<td></td>
</tr>
<tr>
<td>Car</td>
<td>1200 \times -12 = 1200 \ V</td>
<td>1200 \ V</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>-14,400</td>
<td>4,800 \ V</td>
<td></td>
</tr>
</tbody>
</table>

Determine the velocity of the truck and car immediately after the collision. PSYW

\[
\frac{4800}{1200} = \frac{v}{6000} \Rightarrow v = 0.8 \text{ m/s}
\]
Impulse and Momentum Change

An object with momentum must be **moving**. The more momentum an object has, the more difficult it will be to stop the object. Thus, it would require a greater amount of force or a longer amount of time or both to bring such an object to a halt. As the **force** acts upon the object for a given amount of **time**, the object's velocity is changed; and hence, the object's **momentum** is changed.

If the force acts opposite the object's motion, the force **slows** down the object.
If a force acts in the same direction as the object's motion, then the force **speeds up** the object.

Recall, Newton's Second Law states that an unbalanced force causes an object to 

\[ \Delta = \text{change} \]

\[ F = m \times a = m \times \frac{\Delta v}{t} \]

Impulse = Force * time = change in momentum

The above equation is known as the Impulse-Momentum Theorem, where impulse is defined as the force multiplied by time.

Momentum also can be expressed as units of \(Ns\).

**Implications of the Impulse-Momentum Theorem**

First, an impulse acting on an object causes a change in the momentum.

From the Impulse-Momentum Theorem, we can solve for the average force exerted on the object

\[ \text{Force} \times \text{time} = \text{change in momentum} \]

\[ \text{Force} = \frac{\text{change in momentum}}{\text{time of impact}} \]

So for a given amount of change in momentum, the greater the time of impact, the smaller the average force acting on the object.
Simple Computations with Impulse = Momentum Change

Read from Lesson 1 of the Momentum and Collisions chapter at The Physics Classroom:
http://www.physicsclassroom.com/Class/momentum/u4l1b.html
http://www.physicsclassroom.com/Class/momentum/u4l1c.html

A car with a mass of 1000 kg is at rest at a stoplight. When the light turns green, it is pushed by a net force of 2000 N for 10 s.

1. What is the value of the acceleration that the car experiences?
\[ \text{accel} = \frac{\text{net force}}{\text{mass}} = \frac{2000 \text{ N}}{1000 \text{ kg}} = \frac{2 \text{ m/s}^2}{1} \]

2. What is the value of the change in velocity that the car experiences?
\[ \text{change in vel} = \frac{\text{change in momentum}}{\text{time}} = \frac{2 \text{ m/s}}{10 \text{ s}} = \frac{20 \text{ m/s}}{10 \text{ s}} \]

3. What is the value of the impulse on the car?
\[ \text{Impulse} = F \times t = 2000 \text{ N} \times 10 \text{ s} = 20,000 \text{ Ns} \]

4. What is the value of the change in momentum that the car experiences?
\[ \text{Impulse} = \text{change in momentum} = 20,000 \text{ Ns} \]

5. What is the final velocity of the car at the end of 10 seconds?
\[ \text{final vel} = \text{orig vel} + \text{change in vel} = 0 + \frac{20 \text{ m/s}}{10 \text{ s}} = \frac{20 \text{ m/s}}{10 \text{ s}} \]

The car continues at this speed for a while.

6. What is the value of the change in momentum the car experiences as it continues at this velocity?
\[ \text{change in momentum} = 0 \]

7. What is the value of the impulse on the car as it continues at this velocity?
\[ \text{Impulse} = 0 \]

The brakes are applied to the car, causing it to come to rest in 4 s.

8. What is the value of the change in momentum that the car experiences?
\[ \text{change in momentum} = 1000 \text{ kg} \times (0 - 20) = -20,000 \text{ kg m/s} \]

9. What is the value of the impulse on the car?
\[ \text{Impulse} = 0 \]

10. What is the value of the force (average) that causes the car to stop?

11. What is the acceleration of the car as it stops?
There is a disease known as *formula fixation* which is common among physics students. It particularly infects those who perceive physics as an applied math course where numbers and equations are simply combined to solve algebra problems. However, this is not the true nature of physics. Physics concerns itself with ideas and concepts which provide a reasonable explanation of the physical world. When students divorce the mathematics from the ideas, formula fixation takes root and even mathematical problem-solving can become difficult. Do you have *formula fixation*? Test your health by trying these computational problems.

12. A force of 800 N causes a 80-kg fullback to change his velocity by 10 m/s. Determine the impulse experienced by the fullback. PSYW

\[
\text{Impulse} = \Delta \text{momentum} = m \cdot \Delta v = 80 \text{ kg} \cdot (10 \text{ m/s}) = 800 \text{ kg m/s}
\]

13. A 80-kg soccer ball experiences an impulse of 25 N·s. Determine the momentum change of the soccer ball. PSYW

\[
\text{Impulse} = \Delta \text{momentum} = 25 \text{ N·s}
\]

14. A 1200-kg car is brought from 25 m/s to 10 m/s over a time period of 5.0 seconds. Determine the force experienced by the car. PSYW

\[
\text{Force} = m \cdot \Delta \text{velocity} = 1200 \text{ kg} \cdot \left( \frac{10 \text{ m/s} - 25 \text{ m/s}}{5 \text{ sec}} \right) = -3600 \text{ N}
\]

15. A 90-kg tight end moving at 9.0 m/s encounters a 400 N·s impulse. Determine the velocity change of the tight end. PSYW

\[
\text{Impulse} = \text{mass} \cdot \Delta \text{velocity} = 400 \text{ N·s} = 90 \text{ kg} \cdot \Delta v
\]

\[
\Delta v = \frac{400 \text{ N·s}}{90 \text{ kg}} = 4.4 \text{ m/s}
\]

16. A 0.10-kg hockey puck decreases its speed from 40 m/s to 0 m/s in 0.025 s. Determine the force which it experiences. PSYW

\[
\text{Force} = \frac{0.1 \text{ kg} \cdot (-40 \text{ m/s})}{0.025 \text{ s}} = -160 \text{ N}
\]

17. A Real Brain Twister: A 0.10-kg hockey puck is at rest. It encounters a force of 20 N for 0.2 seconds which sets it into motion. Over the next 2.0 seconds, it encounters 0.4 Newtons of resistance force. Finally, it encounters a final force of 24 N for 0.05 seconds in the direction of motion. What is the final velocity of the hockey puck. PSYW

You may have been *tricked*, but those were not intended as trick questions. The questions were intended to test your understanding of the concepts of momentum change, impulse, mass, force, time and velocity change. How is your understanding level progressing? Do you have formula fixation?

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